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Turbulent Momentum Transfer in Two-Phase Cylindrical Couette Flow

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Torque measurements were made in experiments carried out using a cylindrical Couette apparatus with the inner cylinder rotating and the outer one stationary. The fluid systems employed consisted of distilled water, 50% distilled-water glycerine solutions, and these with several concentrations of suspended polystyrene spheres (through 40 on 70 mesh). The parameters employed by Bjorklund and Kays were found to correlate the data successfully when suspension viscosities were used for the two-phase systems.

This paper discusses momentum transfer processes which occur in two-phase turbulent systems of polystyrene spheres in distilled water and 50% distilled water-glycerine solutions flowing between a rotating inner cylinder and a stationary outer one. This geometry has been utilized in the past for studies of transport processes, usually in single-phase systems, so that its utilization as a chemical engineering research tool for studies of more complex multiphase systems has not been fully explored.

The use of a cylindrical Couette flow apparatus as a device for investigating complex two-phase phenomena would appear to have several advantages:

1. The process is confined in a simple, well-defined geometry.
2. Large heat transfer surface to volume ratio ensures good temperature control characteristics.
3. At high rotational velocities, the bulk of the homogeneous phase is practically uniform in temperature and concentration.
4. Minor density differences in the phase present would be ineffective in causing a classification of the phase because of the three-dimensional vortices which exist.

The first item states an obvious advantage over other commonly used devices such as stirred tanks making use of turbine—or propeller—impeller devices. Item 2 also implies an advantage in control and operation over the more standard mixer-tank apparatus. The last two items are tantamount to stating that mixing of the phases takes place very efficiently.

Two disadvantages associated with an apparatus of this type are

1. The fluid flow patterns are very complicated so that the interpretation of transport measurements for predicting phenomena in equipment with more standard geometries becomes a questionable procedure.
2. Effects due to large differences in densities of the phases are difficult to avoid; with the axis of the apparatus vertical, axial gradients of the suspended phase would readily occur.

This second item represents a major disadvantage, and heterogeneous systems in which density differences are large may not be considered for use in the Couette apparatus. However this fails to preclude possible use of this apparatus for such diverse processes as are included in liquid-liquid and many solid-liquid contacting operations.

From the foregoing it is apparent that a justifiable investigation into the general, potential use of the cylindrical Couette flow apparatus as a research tool involves a study of the nature of the fluid mechanics of turbulent flows in the annular space. A stationary outer wall is employed so that glass or a transparent polymeric material is readily used for the outer cylindrical container to enable observations of the interior.

Taylor (*14*) initiated much of the current interest in the cylindrical Couette geometry by predicting the critical speed at which instability sets in. He also demonstrated the existence of Taylor vortices which occur when the critical speed has been exceeded. Later Kaye and Elgar (9) showed that the Taylor number is an especially useful parameter in Couette flow studies. Taylor's expression for the critical speed, given explicitly for the critical Taylor number, with only the inner cylinder rotating, is

$N_{Ta}(CR) =$

$$\left\{\frac{48.70\left(2+\frac{d}{R_0}\right)}{0.0571\left(1-0.652\frac{d}{R_0}\right)+0.00056\Big/\left(1-0.652\frac{d}{R_0}\right)}\right\}^{1/2} \quad (1)$$

This expression was obtained from the summary given in reference *1*. The expression for $N_{Ta}(CR)$ is not very sensitive to d/R_0 for small values of d/R_0. Lewis (*10*) experimentally demonstrated that Taylor's formula is valid for d/R_0 as high as $d/R_0 = 0.71$.

Taylor measured velocity distributions in Couette flow with the inner cylinder rotating at high speeds and, after extrapolating his results to zero size pitot tube, concluded that 80% of the gap width corresponds to a constant value of moment of momentum $u\ r$ (*15*). Pai (*12*) obtained velocity distributions using a hot wire anemometer and concluded that vortices are present at high speeds in the presence of a high level of turbulence. The presence of vortices in turbulent Couette flow was demonstrated and discussed by Kaye and Elgar (*9*).

Bjorklund and Kays (*7*) correlated the single-phase friction data of Taylor (*16*) and Wagner (*19*) with one curve which applied for various radius ratios; that is, they were able to correlate the data so that another parameter, say d/R_0, which should appear explicitly from dimensional considerations alone, does not do so. Their correlation is

for $N_{Ta} < 90$:

$$\frac{fN_{Ta}}{(fN_{Ta})_{\text{lam}}} = 0.0388\ N_{Ta}{}^{0.877} \quad (2)$$

for $N_{Ta} > 90$

$$\frac{fN_{Ta}}{(fN_{Ta})_{\text{lam}}} = 0.19\ N_{Ta}{}^{0.522} \quad (3)$$

and, where

$$(fN_{Ta})_{\text{lam}} = \frac{4(d/R_0)^{3/2}}{1-\left(\frac{1}{1+d/R_0}\right)^2} \quad (4)$$

Equation (4) comes directly from the shear-stress distribution for laminar cylindrical Couette flow and the definition of the friction factor.

$$f = \frac{2\tau_0 g_c}{\rho u_0{}^2} \quad (5)$$

Their correlation does not take into account the increasing trend of friction factor with high Taylor number ($N_{Ta} >$ 10,000) which is evident from the data as given by Bjorklund and Kays.

In addition to the studies noted above there have been other studies of heat transfer and mass transfer to the walls of the Couette apparatus; these include electrode reactions (*3*), dissolution with and without chemical reaction (*13*), reverse osmosis systems (*4*), and heat transfer in air (*15*, *1*) and in liquid systems (*7*). Clay (*2*) studied liquid drop-size distributions in a turbulent Couette system.

In this paper we give experimental results obtained from one- and two-phase friction loss experiments and present the data as suggested by Bjorklund and Kays. The Taylor number range investigated extends to 80,000 and homogeneous liquid and solid-liquid systems are investigated. These data suggest that the two-phase system, with little or no density difference between phases, may be considered as a homogeneous system insofar as gross momentum transport is involved.

APPARATUS

The apparatus consisted of a stainless steel cylindrical rotor, vertically positioned and centered with respect to a torque table. The cylinder was 11 5/16 in. long with a diameter of 5 13/16 ± 0.004 in. The top and bottom of the cylinder consisted of ¼-in. plate and the top plate contained a threaded fitting which could be fastened to the shaft. Both shaft and cylinder were designed to circulate a heat transfer medium for future crystallization studies but this feature is not involved in this discussion. The top and bottom of the cylinder extended to a slightly larger diameter than the main portion of the rotor but end effects were accounted for in the analysis of the data obtained.

The entire rotor assembly could be raised or lowered by a rack and pinion gear arrangement. The cylinder was rotated by a Graham variable-speed drive powered by a ¼-hp. induction motor.

Two stationary outer cylinders were used for this study. One was a Plexiglas cylinder whose inside diameter was 6 11/16 in. and was 12¾ in. long. The bottom consisted of Plexiglas plate. The other cylinder consisted of a stainless steel cylinder whose inside diameter was 7¾ in. and was 13 in. long. The bottom plate was also of stainless steel. Both plates, which served as bottoms, extended beyond the vessel walls so that they could be clamped to the torque table conveniently once the vessel was centered with respect to the rotor. Both cylinders were covered at the top by two well fitting halves of circular Plexiglas plate which was grooved to accommodate both cylindrical vessels. The top contained a ⅝-in. hole for a thermometer and an oversized center opening which accommodated the shaft supporting the rotor.

During ordinary operation of the apparatus, the liquid level was maintained above the rotor when the rotor was stationary—and when it was rotating a vortex formed about the shaft which extended about halfway across the rotor—but the top surface of the rotor was completely submerged. This was found to be a fairly sensitive check on the concentricity of the apparatus; a slight eccentricity caused noticeable eccentricity of the vortex position or an overflow of the air into the annular gap. Although the torque table, container, and rotor and shaft were rigidly connected to the same frame, it was exceedingly difficult to avoid vibrations completely. However, there appeared to be no observable effects of these vibrations upon torque measurements when they occurred. Other details of the apparatus are given in reference *5*.

PROCEDURE

During an experimental run for the determination of the torque, the inner cylinder was rotated at the desired speed. The applied torque required to maintain the outer cylinder stationary was measured by placing weights upon a pan attached by a thread via a pulley to the outside perimeter of the table until the pan just started downward motion, and by removing weights until the pan showed first indications of upward motion. The average of these weight determinations was used to compute the torque. Immediately after each run the temperature of the contents was measured by inserting a thermometer into the annular gap. No torque determinations were made with the system in laminar flow; it is doubtful that acceptable accuracy could have been obtained for such runs with the present system, because the error in a run with a small torque is probably significant as shown by the difference in the two weighings necessary for each torque determination. This is evident in the data used to prepare the results graphically later in this paper, and available in reference *5*.

The end effects for a range of systems and rotational velocities were determined by obtaining torque-liquid level data for about eight or nine levels and then graphing these data and drawing a straight line through the central group of points (usually about seven data points). This was possible for all cases. Comparison of the differences in torques obtained from the graph corresponding to levels of the base and top of the rotor with the torque measurement obtained for the fully submerged rotor provided an estimate of the end effects. These

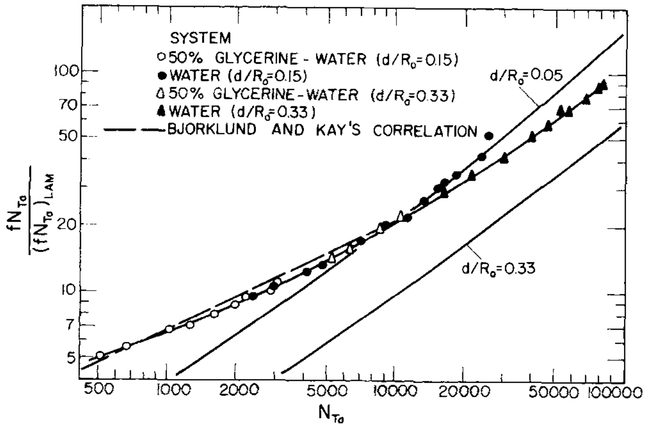

Fig. 1. Friction factor data for homogeneous liquid systems.

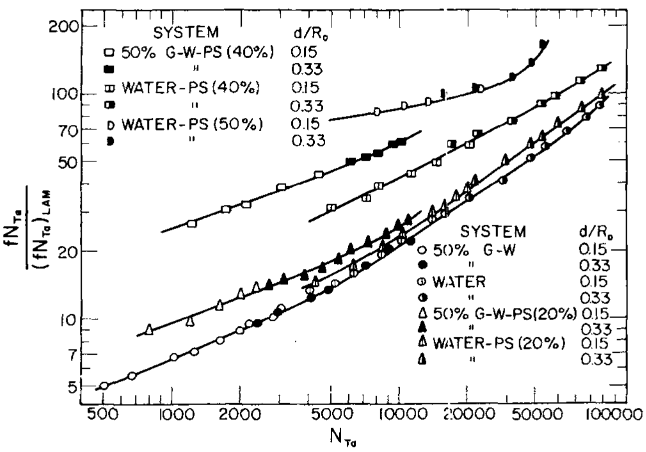

Fig. 2. Friction factor data for solids-liquid suspensions based upon homogeneous system physical properties.

were found to average 15 ± 3% of the fully submerged torque reading for the smaller container ($d/R_0 = 0.15$) and 17 ± 3% for the larger ($d/R_0 = 0.33$).

Filling the apparatus with either homogeneous liquid or slurry systems was carried out in increments. Between incremental additions the rotor was rotated to remove air bubbles trapped in the system. A drain was provided on each of the cylindrical containers so that disturbing the position of the components for emptying and refilling purposes was seldom necessary. Further details of procedure may be obtained from reference 5.

The systems used were distilled water and 50% glycerine-water solution [density 1.058 g./ml. and viscosity 4.21 cp. (30°C.)]. Polystyrene spheres, with over 80% passing through 40 and retained on 70 mesh (density 1.03 ± 0.02 g./ml.), were used to provide solid-liquid mixtures with the above liquids.

RESULTS

The experimental friction factor data for pure fluids in turbulent Couette flow for two clearance ratios (d/R_0 = 0.15 and 0.33) are plotted in Figure 1 in terms of the coordinate variables suggested by Bjorklund and Kays (*1*). Two liquids, water and a 50% glycerine-water solution, were used. These differ significantly in viscosity and only slightly in density from each other and the polystyrene spheres. Figure 1 also compares the experimental results with the correlation of Bjorklund and Kays given as Equation (3).

Figure 2 gives friction factor data for suspensions of 20, 40, and 50 wt. % (polystyrene) solids in water and 20 and 40% solids in glycerine-water (50%) solution. The viscosities and densities of the pure liquids were used in the computation of the Taylor numbers for these plots.

DISCUSSION

Figure 1 graphically presents the results of calculations for the friction factor based on the eddy viscosity notion. These theoretical curves show dependence upon the parameter d/R_0, and d/R_0 = 0.05 and 0.33 are displayed. In the low Reynolds number region (not shown) these graphs intersect that for laminar flow $[(fN_{Ta})/(fN_{Ta})_{\text{lam}} = 1]$ at the critical Taylor number as given by Equation (1). This calculation is based on the assumption of a nonzero time average angular flow, other average components being zero, and the distribution of eddy viscosity according to Gill and Scher (*6*). The derivation is given in reference 5.

The computed curves based on theory do not take account of the presence of vortices. It appears that as the Taylor number increases, the turbulence characterized by random motions becomes more predominant as the mechanism by which transport processes take place and the trend of the curves based on theory and the data are similar. Thus it might be expected that the parameter d/R_0 might become significant for Taylor numbers larger than those for which data have been obtained. That this may well be true is further suggested by the apparent deviation of data for the different d/R_0 ratios in Figure 1 at Taylor numbers of 20,000 and greater. Furthermore, this may be the reason for an apparent difficulty in correlating the data of Taylor as illustrated in reference *1*. However the complete disappearance of vortices at higher Taylor numbers is not in accordance with the observations of Elgar and Kaye (*9*). Also, for all friction factor measurements for d/R_0 = 0.15 obtained in this work, which are shown in Figure 2, the outer cylinder was transparent and the vortex motion was visible; no obvious major changes in its structure occurred over the entire range of Taylor numbers. Thus despite the appearance of the theoretical curves in Figure 1, there is no evidence that vortices do not occur at very high Taylor number values. The deviations in higher Taylor number data, from each other, observed in Figure 1 may be due to the mechanical vibrations which were more difficult to avoid in this speed range.

The application of the equation based on one-dimensional theory to the geometry $d/R_0 \to \infty$ and graphing results as log f vs. log $\left(\frac{u_0 R_0 \rho}{\mu}\right)$ yields a curve which is lower than the experimental results of Theodorsen and Regier (*17*), again suggesting the presence of vortices. The comments of Marangozis and Johnson (*11*) are pertinent to any discussion of the requirement of a parameter d/R_0; in the discussion of mass transfer in a Couette system they argued for the presence of more than one distance parameter d or R_0. Previous workers (*3*) had made use of only R_0, thus prompting the remarks in reference *11*.

Because of the presence of secondary flows to the extent indicated above, it is interesting to query how suspended solids affect the momentum transfer process. The viscosities were recalculated on the basis of the work of Thomas (*18*), who carefully and critically analyzed the published data on the viscosity of suspensions. Thomas' equation is

$$\frac{\mu_s}{\mu_0} = 1 + 2.5\phi + 10.05\phi^2 + 0.00273 \exp(16.6\phi) \quad (6)$$

in which ϕ is the volume fraction of solids. These results are shown in Figure 3.

Comparing these data with those obtained for the homo-

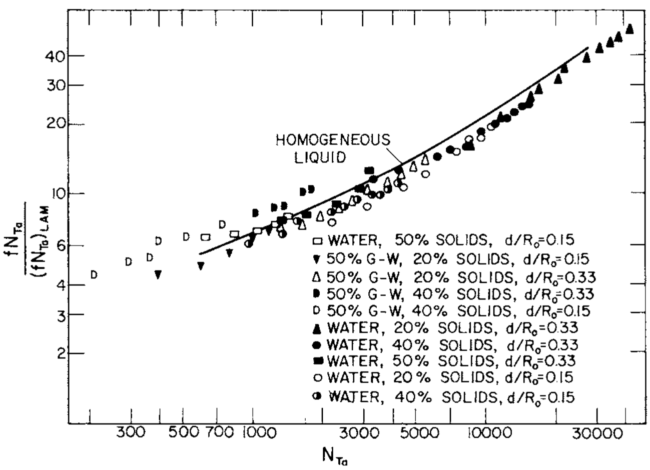

Fig. 3. Friction factor data for solids-liquid suspensions based upon suspension viscosities and densities.

geneous systems, which are shown as the curve in Figure 3, one may conclude that, at least for the systems employed here, physical property modifications do bring together the two sets of data, for heterogeneous and homogeneous systems, except for those runs involving very large concentrations of solids. The friction factors obtained for 40 wt. % solids (39.2 vol. %) suspended in a 50% glycerol solution are uniformly higher than but parallel to the homogeneous line; the friction factors for the 50 wt. % solids suspended in water are much less uniform, although they also show a generally higher friction factor than the corresponding homogeneous value. The behavior of these latter data for the lower rotational speed runs may have been brought about by a classification of the particles due to density differences in the gravitational field, but this effect diminished as speed (and turbulent mixing effects) was increased. Runs had been attempted with sand suspended in the liquid phase (over 90% through 100 mesh) but were abandoned because of the poor suspension characteristics of the system. This was probably not entirely the reason for the high data appearing in the runs involving glycerol solution—40% solids runs (40.6 vol. %)—as the settling velocities in the more viscous solution are substantially less than they are in water.

A possible explanation for the uniform deviation from the homogeneous fluid lines of the high solids concentration runs for the glycerol system may be due to the classification of the lighter solids in the centrifugal field. The solids would tend to concentrate in the vicinity of the moving inner surface. The near laminar flow region in the vicinity of the wall is large enough to accommodate several layers of the solids used in this study. A slight concentration increase would effect a relatively large increase in friction factor due to the increased suspension viscosity. This is apparent from Thomas' equation, which shows that an increment in ϕ of 0.01 causes a change in Taylor number of 8% for the concentrated glycerol suspension. A similar increment in concentration in the 50% water suspension results in a larger effect upon the Taylor number—12%. But here the particles are heavier than the surrounding liquid and concentration increases at the outer wall would probably have less effect upon the friction factor. Additional evidence supporting the importance of density variation in this type of flow system is given by the results of Ho, Nardacci, and Nissan (*8*), who used the Grashof number to treat their heat transfer results, indicating the sensitivity of the behavior of the Couette apparatus to density differences.

CONCLUSIONS

The parameters employed by Bjorklund and Kays to describe turbulent cylindrical Couette flow momentum transport are successfully employed for correlating friction factor data over a large range of Taylor numbers and for heterogeneous systems. These present results for heterogeneous systems are probably restricted to phases of equal densities. Comparison of these friction factor data with those suggested by classical, rectilinear geometry turbulence suggests that turbulence contributes more to the transport with increasing Taylor number than do the vortices, which, nevertheless, persist at high Taylor numbers.

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NOTATION

d = gap width, ft.
f = Fanning friction factor, $2\tau_0 g_c/\rho u_0^2$, dimensionless
g_c = gravitational constant, (lb.$_m$/lb.$_f$)/(ft.)(sec.)(sec.)
N_{Ta} = Taylor number, $u_0 d\rho/\mu\ (d/R_0)^{1/2}$, dimensionless
$N_{Ta}(CR)$ = critical value of the Taylor number as given by Equation (1)
R_0 = inner cylinder radius, ft.
u_0 = linear velocity of surface of inner cylinder, ft./sec.

Greek Letters

ϕ = volume fraction of suspended solids, cu.ft. solids/cu.ft. suspension
ρ = fluid density, lb.$_m$/cu.ft.
τ_0 = shear stress at inner wall, lb.$_f$/sq.ft.
μ_0 = viscosity of homogeneous fluid, lb.$_m$/(ft.)(sec.)
μ_s = viscosity of suspension, lb.$_m$/(ft.)(sec.)

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